

問) $x^5 + \frac{1}{x^5} = \frac{205}{16}(x + \frac{1}{x})$ をみたす x を求めよ。

$$\begin{aligned} & (x + \frac{1}{x})^5 \\ &= x^5 + 5 \cdot x^4 \cdot \frac{1}{x} + 10x^3 \cdot \frac{1}{x^2} + 10x^2 \cdot \frac{1}{x^3} + 5x \cdot \frac{1}{x^4} + \frac{1}{x^5} \\ &= x^5 + 5 \cdot x^3 + 10x + 10 \cdot \frac{1}{x} + 5 \cdot \frac{1}{x^3} + \frac{1}{x^5} \\ &= x^5 + \frac{1}{x^5} + 5(x^3 + \frac{1}{x^3}) + 10(x + \frac{1}{x}) \\ &= x^5 + \frac{1}{x^5} + 5(x + \frac{1}{x})^3 - 15(x + \frac{1}{x}) + 10(x + \frac{1}{x}) \\ &x^5 + \frac{1}{x^5} = (x + \frac{1}{x})^5 - 5(x + \frac{1}{x})^3 + 5(x + \frac{1}{x}) \text{ より} \\ &(x + \frac{1}{x})^5 - 5(x + \frac{1}{x})^3 + 5(x + \frac{1}{x}) - \frac{205}{16}(x + \frac{1}{x}) = 0 \\ &x + \frac{1}{x} = t \text{ とおくと } t^5 - 5t^3 - \frac{125}{16}t = 0 \\ &t(16t^4 - 80t^2 - 125) = 0 \text{ より } t(4t^2 + 5)(4t^2 - 25) = 0 \\ &t \text{ は } 0 \text{ 以外の実数なので、 } t = \pm \frac{5}{2} \\ &x + \frac{1}{x} = \pm \frac{5}{2} \text{ より } \underline{x = \pm 2, \pm \frac{1}{2}} \end{aligned}$$