

問) $\sqrt{\sqrt[3]{3} + \frac{2}{\sqrt[3]{3}-1}}$ の値を求めよ。

$$a = \sqrt{\sqrt[3]{3} + \frac{2}{\sqrt[3]{3}-1}} \quad , \quad t = \sqrt[3]{3} \quad \text{とおく。}$$

$$a = \sqrt{t + \frac{2}{t-1}} = \sqrt{\frac{t^2-t+2}{t-1}}$$

$$a^2 = \frac{t^2-t+2}{t-1} = \frac{(t^2-t+2)(t^2+t+1)}{(t-1)(t^2+t+1)} = \frac{(t^2-t+2)(t^2+t+1)}{t^3-1}$$

$$= \frac{t^4+t^3+t^2-t^3-t^2-t+2t^2+2t+2}{t^3-1} = \frac{t^4+2t^2+t+2}{t^3-1}$$

$$t^3 = 3 \quad \text{より}$$

$$a^2 = \frac{3t+2t^2+t+2}{3-1} = t^2 + 2t + 1 = (t + 1)^2$$

$$a > 0 \quad \text{より} \quad a = t + 1 = \sqrt[3]{3} + 1$$